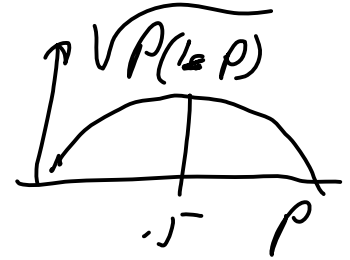


STAT 200 4-6-09a

CHECK \hat{p} EXAMPLE



car-buying public? We at Kelley Blue Book asked 500 potential new-vehicle buyers that yield answers to that multi-billion dollar question. Their

Asked "How likely would you be to buy a car from General Motors if they were to go bankrupt," 42.1 percent of new-vehicle buyers said they were either "not at all likely" or "not very likely" to do so.

SUMMARY, KELLY BLUE BOOK SURVEY OF 500 "POTENTIAL NEW VEHICLE BUYERS." ASKED "HOW LIKELY TO BUY GM VEH. IF GM GOES BANKRUPT." A POSSIBLE RESPONSE IS "NOT LIKELY" TO WHICH 42.1 ANSWER.

NAIVE Z-BASED CI FOR p : $\hat{p} \pm z \sqrt{\hat{p}\hat{q}}/\sqrt{n}$

e.g. $P(p \text{ "in"} \hat{p} \pm 1.96 \sqrt{\hat{p}\hat{q}}/\sqrt{n}) \rightarrow .95 \text{ as } n \rightarrow \infty$.

SAMPLES EQUAL PROBABILITY and WITH REPLACEMENT

A NAIVE CI FOR FRACTION p (ANS. "NOT LIKELY TO BUY")

ESTD $\hat{p} = .421$ 95% CI FOR p : $.421 \pm 1.96 \frac{\sqrt{.421 \cdot .579}}{\sqrt{500}}$

IGNORING FPC $\sqrt{\frac{N-n}{N-1}}$

ESTD \hat{p} \rightarrow $\approx 1.96 \cdot \frac{1}{\sqrt{500}}$
MOST OF \hat{p} \rightarrow ≈ 22.36

- Commentary: GM chief sacked due to AIG woes
- In Depth: Commentaries

Even with a warranty guarantee from the federal consumers are not very enthusiastic about considering such a guarantee only moves the needle about 10

points with 39.8 percent saying they were either "not at all likely" or "not very likely" to purchase a vehicle in such a situation.

W/O GOVT WARRANTY BACKUP $\hat{p} = .421$ "NOT LIKELY TO BUY"
 ② WITH " " " " $\hat{p} = .398$ "

NAIVE Z-BASED 95% CI FOR p : $.398 \pm 1.96 \frac{\sqrt{.398 \cdot .602}}{\sqrt{500}}$

Difference? $.421 - .398$ PICKED UP (SCALE 0-1) $.023$

TROUBLE CI (NAIVE ONE) REQUIRES CLT - AND WILL NOT BE SO ACCURATE (V/S .95) IF n TOO SMALL OR $p \approx 0$ OR $p \approx 1$. SINCE $\hat{p} = .023 \approx 0$ WE ARE CONCERNED THAT NAIVE CI MAY BE "NOT DOING THE JOB AS PROMISED."

Asked, "How likely would you be to buy a car from General Motors if they were allowed to renegotiate their contracts and agreements with suppliers and laborers?" the
 Given that scenario, only 31.6 percent of new-vehicle buyers said they were either "n
 very likely" to purchase a GM vehicle, and they were out-numbered by the 41.5 perce
 either "very likely" or "extremely likely" to do so.

ASIDE 9498 CH19 DISCUSS WILSON 1927 IMPROVEMENT
 ON NAIVE CI - BETTER HANDLE $n \times \infty$ $p \sim 0$ or $p \sim 1$
 TAKEN UP IN 1998 BY AGRESTI & COULL - SIMPLIFIED.

IDEA: GOT $\hat{p} = .023$ ($n = 500$) ≈ 11 or 12 of 500

EFAD: Suppose $n = 500$ FOUND $X = 12$ "SAVED"
 BY GOVT 'WARRANTY BACKUP.' THEN $\hat{p} = \frac{12}{500} = .024$

1 STEAD A-C SUGGEST FORMING \tilde{n} (n TILDE) $= n + 4$
 AND $\hat{p} = \frac{X+2}{\tilde{n}}$ $\tilde{q} = \frac{n-X+2}{\tilde{n}}$ PRESENT CASE,

$\tilde{n} = 500 + 4 = 504$, $\hat{p} = \frac{12+2}{504} = \frac{14}{504} = .028$, $\tilde{q} = .972$
 THEN GIVE "95% CI": $\hat{p} \pm 1.96 \sqrt{\hat{p}\tilde{q}/\tilde{n}} = .028 \pm 1.96 \frac{\sqrt{.028 \cdot .972}}{\sqrt{504}}$

BETTER CI EXIST FOR p .

RETURN TO NAIVE z -BASED CI FOR p .

$$\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}\hat{q}}}{\sqrt{n}} \quad \text{ESTD m.o.e.}$$

SAY $n=500$

AND $\text{m.o.e.} = .02$ (JUST SAY)

CI $\hat{p} \pm .02$ REALLY WANT LIKE $\hat{p} \pm .005$

ONLY WAY TO GET IT IS

(a) IMPROVE UPON \hat{p} .

OR (b) INCREASE n (MAYBE $\sqrt{\hat{p}\hat{q}}$ WON'T CHANGE TOO MUCH)

SIMPLY EQUATE $1.96 \frac{\sqrt{\hat{p}\hat{q}}}{\sqrt{n}} \leftarrow \text{KNOWN}$ $= .005$ PRECISION YOU WANT -
SOLVE FOR n .

CH19
3abcd

STT 200 4-6-09b

car-buying public? We at Kelley Blue Book asked 500 potential new-vehicle buyers questions that yield answers to that multi-billion dollar question. Their

CH 19:
LOOK AT 3abcd
FOR RECITATION

Asked "How likely would you be to buy a car from General Motors if they were to go bankrupt," 42.1 percent of new-vehicle buyers said they were either "not at all likely" or "not very likely" to do so.

KELLY BLUE BOOK SURVEY "SAMPLE" OF 500 LIKELY CAR BUYERS
ASK ① IF GM GOES BANKRUPT "HOW LIKELY YOU BUY FROM GM?" POPULATION NOT CLEARLY IDENTIFIED.
ANS. 42.1% SAY "NOT AT ALL LIKELY" OR "NOT VERY LIKELY TO."

NAIVE 95% CI FOR p = FRACTION OF POP RESPONDING "NOT LIKELY"

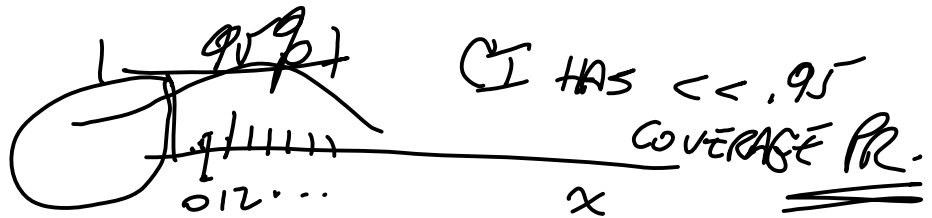
$$\hat{p} \pm (z) 1.96 \sqrt{\hat{p}\hat{q}} / \sqrt{n} = .421 \pm 1.96 \frac{\sqrt{.421 \cdot .579}}{\sqrt{500}}$$

CLAIM: $P(\text{POP MEAN } p \text{ IN } \hat{p} \pm 1.96 \frac{\sqrt{\hat{p}\hat{q}}}{\sqrt{n}}) \approx .95$

BUT MAY FAIL (V.S. 95%) IF n TOO OR $p \approx 0$ OR $p \approx 1$.

- Commentary: GM chief sacked due to AIG woes
- In Depth: Commentaries

points with 39.8 percent saying they were either "not at all likely" or "not very likely" to vehicle in such a situation.



Even with a warranty guarantee from the federal consumers are not very enthusiastic about consic Such a guarantee only moves the needle about t

② (SAME 500 PEOPLE) IF GM GOES BANKRUPTCY + GOVT BACKS WARRANTY THEN NEG RESPONSE DROPS FROM 42.1% TO 39.8% CHANGE IS $.421 - .398 = .023$

CALL THIS $\hat{p} = .023 \sim 0$ TROUBLE?

PEOPLE WHOSE NEG OPINION WAS TURNED.

PP 494-495 SEE THERE ABRESTI - COULL-WILSON 1998 (1927)

REMEDY OFFERED:

.023 of 500

2.3 of 100

2.3×5 of 500

SAV THEY HAD 12 = X RESPONDING NEG OF 500 PROPOSE TO $X+2 = 12+2$ ALSO $n = 500 + 4 = 504$ DEFINE $\tilde{p} = 14/504$ $\hat{\delta} = 490/504$ P-TILDE

SIMPLIFY

RECAP. FOUND $X=12$ "TURNED OPINION" IN 500

$$\hat{p} = \frac{X}{n} = \frac{12}{500} \text{ LEADS TO } \hat{p} \sim 0.$$

REMEDY "ADD 2 (FAKE RESPONSES) TO \hat{p} & \hat{q} GROUPS -

TILDE $\rightarrow \tilde{p} = \frac{14}{504}, \tilde{q} = \frac{490}{504} \quad \tilde{n} = 504$

THEN, IT IS SUGGESTED, JUST DO "REG" CI.

TO SAY - $\tilde{p} \pm 1.96 \frac{\sqrt{\tilde{p}\tilde{q}}}{\sqrt{504} \leftarrow \tilde{n}}$

CLAIM $\Rightarrow P(\hat{p} \text{ "IN" ABOVE CI}) \approx .95$ EVEN THOUGH \hat{p} (\tilde{p} AS WELL)

JUDGE METHOD BY PERFORMANCE $\sim 0.$

** THERE ARE BETTER METHODS **

ANOTHER ISSUE PRECISION



ILLUSTRATE W/ NAIVE METHOD

$\hat{p} \pm 1.96 \frac{\sqrt{\hat{p}\hat{q}}}{\sqrt{n}}$ — ESTD MOE OF \hat{p}

PRECISION — WANT IT TO BE ~ 0

SUPPOSE YOU FIND FOR $n=500$ $\hat{p} = .421$

PRECISION IS $\frac{1.96 \sqrt{.421 \cdot .579}}{\sqrt{500}} \approx \frac{2 \sqrt{\frac{1}{2} \cdot \frac{1}{2}}}{\sqrt{500}} \approx 22$

SAY $\approx .05$

PRECISION

SAY NOT GOOD ENOUGH! I WANT PRECISION .001

TO GET PRECISION .001

$$\hat{p} \pm .001$$

SOLVE FOR n

$$\frac{1.96 \sqrt{p^2 q^2 n}}{\sqrt{n}}$$

VARY $\rightarrow \sqrt{n}$
(INCREASE)

$$= .001$$

WANT

SOLVE

$$n = \left(\frac{1.96 \sqrt{.421579}}{.001} \right)^2$$

1000000